

# WP8: Multi-Criteria Decision Support System

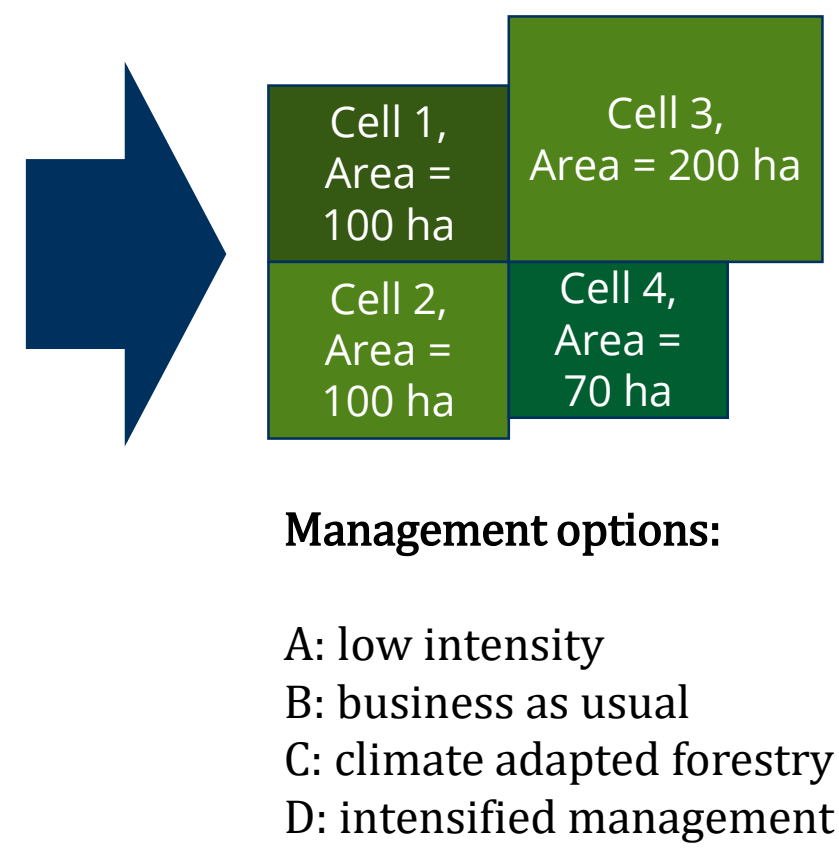
Prof. Dr. Udo Buscher, Maria Beranek, Michael Hölscher  
TU Dresden, Germany

## Introduction

WP8 develops a Multi-Criteria Decision Support System (MCDSS), which provides information to stakeholders on the forest-wood value chain based on a combination of (stakeholder) objectives and quantified impacts. The aimed solutions will harmonize the interests of all stakeholders by assuming a virtual superior decision maker who is optimizing an overall objective (sum of all stakeholder objectives). The MCDSS serves to achieve resilient forest production systems. Actors will be empowered to understand the far-reaching consequences of their decisions in a multicriteria decision-making environment.

Initial stand	Species 1 share	Species 2 share	...
1	10	10	...
2	10	10	...
3	50	50	...
...	...	...	...
...	0	100	...

Initial Stand	Management option	Indicators = 20							
1	A	0	1	1	...	5	...	...	...
1	B	1	0	1	...	0	...	...	...
1	C	1	1	1	...	1	...	...	...
1	D	1	2	0	...	0	...	...	...
2	A	...	...	...	...	...	...	...	...
2	B	...	...	...	...	...	...	...	...
2	C	...	...	...	...	...	...	...	...
2	D	...	...	...	...	...	...	...	...
...	A	...	...	...	...	...	...	...	...
...	B	...	...	...	...	...	...	...	...
...	C	...	...	...	...	...	...	...	...
...	D	...	...	...	...	...	...	...	...



Cell	Management option
1	A
2	B
3	A
4	D

## Methodology

### Mathematical Model

Maximize weighted sum objective:

$$\max y_{o,g}^* = \frac{y_{o,g} - Y_{o,g}}{Y_{o,g} - Y_{o,g}} \quad \forall g \in G, o \in O_g$$

$$y_{(i,free),g} = v_{i,g} \quad \forall g \in G, i \in I_g^{free}$$

Indicator constraints:

$$\text{harvested timber} \quad v_{(harvested\ timber),g} = \sum_{t \in T} \sum_{c \in C_g} \sum_{m \in M} x_{c,m} \cdot B_{c,m,t}^{HarvestedTimber} \cdot F_c \quad \forall g \in G$$

$$\text{periodic annual increment} \quad v_{(periodic\ annual\ increment),g} = \sum_{t \in T} \sum_{c \in C_g} \sum_{m \in M} x_{c,m} \cdot B_{c,m,t}^{PeriodicAnnualIncrement} \quad \forall g \in G$$

$$\text{Carbon-sequestration} \quad v_{(c-sequestration),g} = \sum_{t \in T} \sum_{c \in C_g} \sum_{m \in M} x_{c,m} \cdot B_{c,m,t}^{Carbon} \cdot F_c \quad \forall g \in G$$

$$\text{Standing timber} \quad v_{(standing\ timber),g} = \sum_{t \in T} \sum_{c \in C_g} \sum_{m \in M} x_{c,m} \cdot B_{c,m,t}^{Standing} \cdot F_c \quad \forall g \in G$$

$$\text{Large trees} \quad v_{(large\ trees),g} = \sum_{t \in T} \sum_{c \in C_g} \sum_{m \in M} x_{c,m} \cdot B_{c,m,t}^{Trees} \cdot F_c \quad \forall g \in G$$

$$\dots$$

$$\text{Shannon Index} \quad v_{(shannon\ index),g} = - \sum_{t \in T} \sum_{c \in C_g} \sum_{m \in M} \sum_{s \in S} x_{c,m} \cdot F_{c,m,s,t} \cdot \ln(x_{c,m} \cdot F_{c,m,s,t}) \quad \forall g \in G$$

=> Approximation through linearization necessary!

Assumption: shannon index reaches the maximum if following statement applies:

$$\sum_{c \in C_g} \sum_{m \in M} x_{c,m} \cdot F_{c,m,s,t} = \frac{1}{|S|} \quad \forall t \in T, s \in S$$

$$\min \hat{y} = \sum_{t \in T} \sum_{s \in S} (u_{t,s} + o_{t,s})$$

$$\text{s.t.} \quad \sum_{c \in C_g} \sum_{m \in M} x_{c,m} \cdot F_{c,m,s,t} + u_{t,s} - o_{t,s} = \frac{1}{|S|} \quad \forall s \in S, t \in T$$

$$u_{t,s} \leq s_{t,s} \quad \forall s \in S, t \in T$$

$$o_{t,s} \leq 1 - s_{t,s} \quad \forall s \in S, t \in T$$

$$0 \leq o_{t,s}, u_{t,s} \leq 1 \quad \forall s \in S, t \in T$$

$$s_{t,s} \in \{0, 1\} \quad \forall s \in S, t \in T$$

Selecting one management option for each cell

$$\sum_{m \in M} x_{c,m} = 1 \quad \forall c \in C$$

$$x_{c,m} \in \{0, 1\} \quad \forall c \in C, m \in M$$

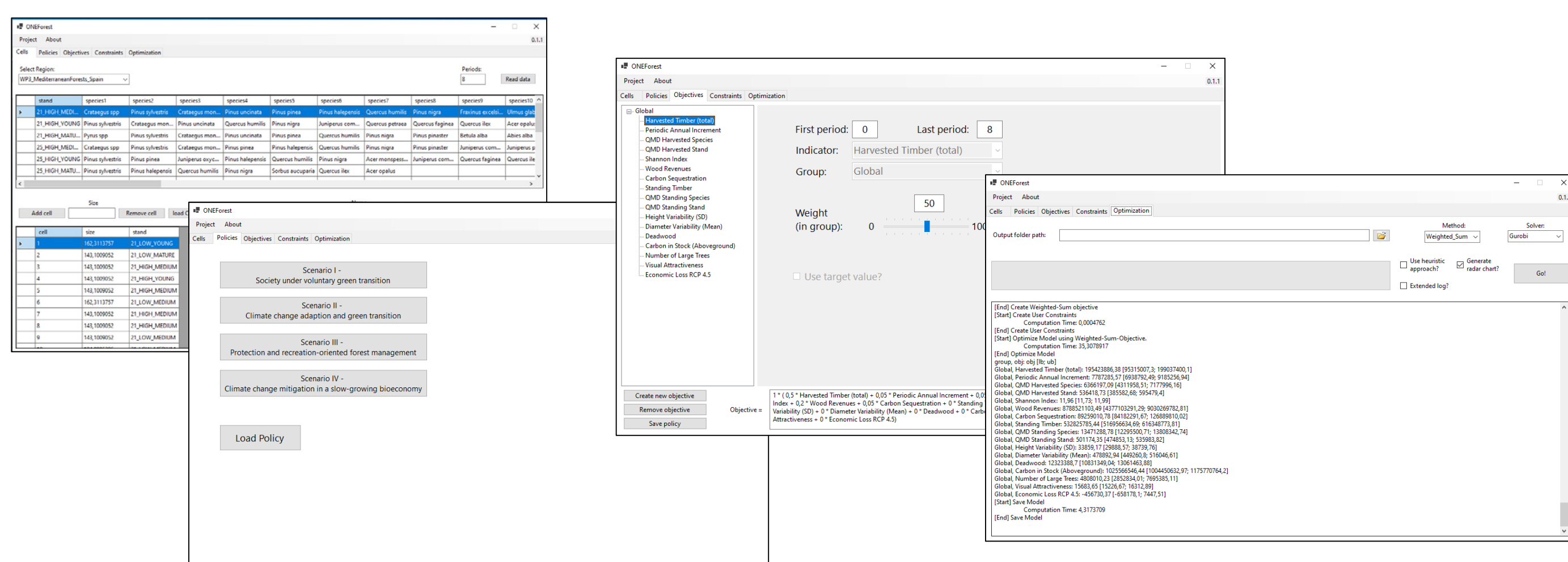
Hard constraints, e.g.

$$\sum_{c \in C_g} \sum_{m \in M} x_{c,m} \cdot B_{c,m,t}^{Trees} \leq Limit \quad \forall t \in T$$

## Prototype of the MCDSS

Main features:

- Replication of forest areas on the basis of four representative case study regions (Estonia, Switzerland, Spain, Germany)
- Consideration of up to 20 indicators, which can be weighted individually in a weighted sum objective
- Utilization of user constraints and target values
- Optimization based on a Mixed Integer Linear Program (MILP)
- Extensive numerical output and graphical analysis with radar charts



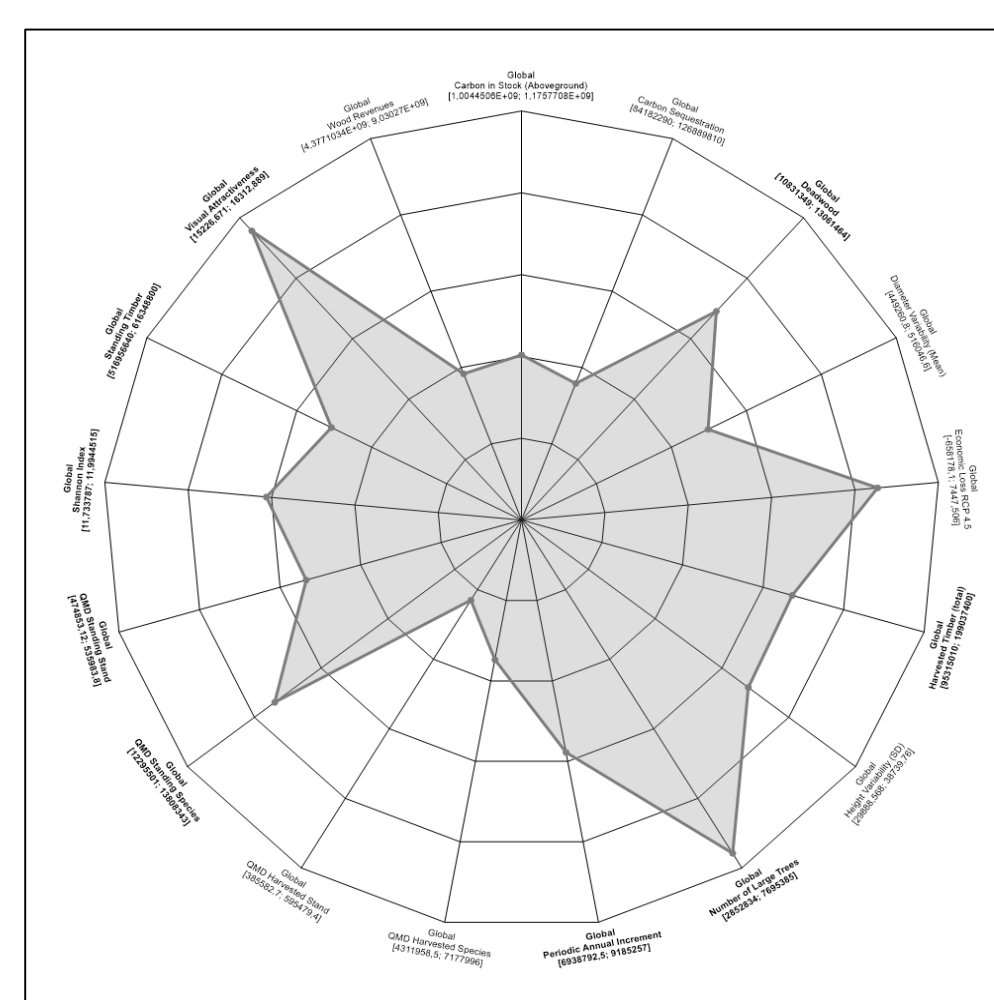
Indices				
t - index of the period	c - index of the cell	g - index of a group	m - index of management option	s - index of species
Sets				
M	Management options	C	Cells	
I <sup>free</sup>	Indicators / considered as objective in group g	C <sub>g</sub>	Cells in group g	O <sub>g</sub> objectives o considered in group g
T	Periods	G	Groups	
Input-Parameters				
Y <sub>o,g</sub>	lower bound for objective o in group g			
Y <sub>o,g</sub> <sup>u</sup>	upper bound for objective o in group g			
B <sub>c,m,t</sub> <sup>HarvestedTimber</sup>	Harvested amount of timber in m <sup>3</sup> of species s of assortment a in period t if management option m is applied in cell c			
B <sub>c,m,t</sub> <sup>PeriodicAnnualIncrement</sup>	Volume increment per period t for cell c with management option m			
B <sub>c,m,t</sub> <sup>Carbon</sup>	Carbon sequestration in period t if management option m is applied in cell c			
B <sub>c,m,t</sub> <sup>Standing</sup>	Standing timber in period t if management option m is applied in cell c			
B <sub>c,m,t</sub> <sup>Trees</sup>	Number of large trees in period t if management option m is applied in cell c			
F <sub>c</sub>	Size of cell c in hectare			
Output-Variables				
Y <sub>o,g</sub> <sup>*</sup>	normalized value of objective o in group g			
v <sub>i,g</sub>	value of indicator i in group g			
x <sub>c,m</sub>	= 1, if management option m is applied for cell c, otherwise 0			

## Results

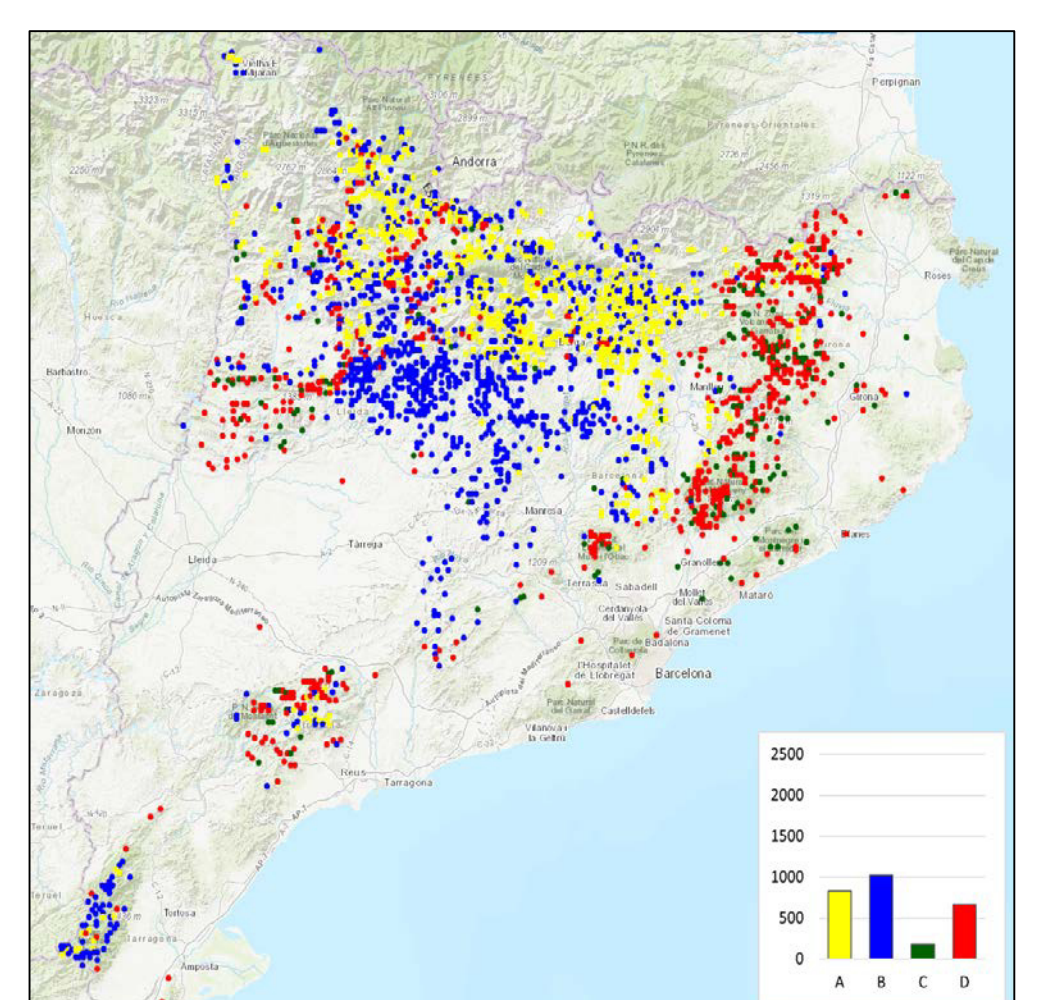
The mathematical model attempts to realize the prioritization of indicators based on the entered weights in the best possible way. As a result of the optimization the user gets an assignment of a management option to each of the cells created. For each indicator the lower and upper bound are calculated and its specific value that results out of the chosen management options. This can be analyzed in a radar chart which is part of the output. For the CSR Catalonia the geo-coordinates of the cells were known, enabling the distribution of the management options to be displayed graphically on a map.

Objective	Value	Lower bound	Upper bound
1. Completion time	60.000000	60.000000	60.000000
2. Shannon Index	11.707088	11.044113	12.000000
3. PeriodicAnnualIncrement	10224.0173	10111.8885	10336.1458
4. Carbon	20204.0476	20044.1133	20363.9811
5. Standing	10014.0476	10004.1133	10023.9811
6. Trees	42614.1173	42504.2220	42724.0126
7. QM1 Standing Stand	122500.75	120004.24	125000.75
8. QM2 Standing Stand	100000.75	97500.75	102500.75
9. Harvested Timber (Total)	301300.75	299370.62	303230.88
10. Periodic Annual Increment	401300.75	399370.62	403230.88
11. Carbon Sequestration	411300.75	409370.62	413230.88
12. Standing Timber	401300.75	399370.62	403230.88
13. Number of Large Trees	200130.75	198100.75	202160.75
14. Carbon sequestration (Total)	401300.75	399370.62	403230.88
15. Standing timber (Total)	401300.75	399370.62	403230.88
16. Number of Large Trees (Total)	200130.75	198100.75	202160.75
17. Carbon sequestration (Total)	401300.75	399370.62	403230.88
18. Standing timber (Total)	401300.75	399370.62	403230.88
19. Number of Large Trees (Total)	200130.75	198100.75	202160.75
20. Carbon sequestration (Total)	401300.75	399370.62	403230.88
21. Standing timber (Total)	401300.75	399370.62	403230.88
22. Number of Large Trees (Total)	200130.75	198100.75	202160.75
23. Carbon sequestration (Total)	401300.75	399370.62	403230.88
24. Standing timber (Total)	401300.75	399370.62	403230.88
25. Number of Large Trees (Total)	200130.75	198100.75	202160.75
26. Carbon sequestration (Total)	401300.75	399370.62	403230.88
27. Standing timber (Total)	401300.75	399370.62	403230.88
28. Number of Large Trees (Total)	200130.75	198100.75	202160.75
29. Carbon sequestration (Total)	401300.75	399370.62	403230.88
30. Standing timber (Total)	401300.75	399370.62	403230.88
31. Number of Large Trees (Total)	200130.75	198100.75	202160.75
32. Carbon sequestration (Total)	401300.75	399370.62	403230.88
33. Standing timber (Total)	401300.75	399370.62	403230.88
34. Number of Large Trees (Total)	200130.75	198100.75	202160.75
35. Carbon sequestration (Total)	401300.75	399370.62	403230.88
36. Standing timber (Total)	401300.75	399370.62	403230.88
37. Number of Large Trees (Total)	200130.75	198100.75	202160.75
38. Carbon sequestration (Total)	401300.75	399370.62	403230.88
39. Standing timber (Total)	401300.75	399370.62	403230.88
40. Number of Large Trees (Total)	200130.75	198100.75	202160.75
41. Carbon sequestration (Total)	401300.75	399370.62	403230.88
42. Standing timber (Total)	401300.75	399370.62	403230.88
43. Number of Large Trees (Total)	200130.75	198100.75	202160.75
44. Carbon sequestration (Total)	401300.75	399370.62	403230.88
45. Standing timber (Total)	401300.75	399370.62	403230.88
46. Number of Large Trees (Total)	200130.75	198100.75	202160.75
47. Carbon sequestration (Total)	401300.75	399370.62	403230.88
48. Standing timber (Total)	401300.75	399370.62	403230.88
49. Number of Large Trees (Total)	200130.75	198100.75	202160.75
50. Carbon sequestration (Total)	401300.75	399370.62	403230.88
51. Standing timber (Total)	401300.75	399370.62	403230.88
52. Number of Large Trees (Total)	200130.75	198100.75	202160.75
53. Carbon sequestration (Total)	401300.75	399370.62	403230.88
54. Standing timber (Total)	401300.75	399370.62	403230.88
55. Number of Large Trees (Total)	200130.75	198100.75	202160.75
56. Carbon sequestration (Total)	401300.75	399370.62	403230.88
57. Standing timber (Total)	401300.75	399370.62	403230.88
58. Number of Large Trees (Total)	200130.75	198100.75	202160.75
59. Carbon sequestration (Total)	401300.75	399370.62	403230.88
60. Standing timber (Total)	401300.75	399370.62	403230.88
61. Number of Large Trees (Total)	200130.75	198100.75	202160.75
62. Carbon sequestration (Total)	401300.75	399370.62	403230.88
63. Standing timber (Total)	401300.75	399370.62	403230.88
64. Number of Large Trees (Total)	200130.75	198100.75	202160.75
65. Carbon sequestration (Total)	401300.75	399370.62	403230.88
66. Standing timber (Total)	401300.75	399370.62	403230.88
67. Number of Large Trees (Total)	200130.75	198100.75	202160.75
68. Carbon sequestration (Total)	401300.75	399370.62	403230.88
69. Standing timber (Total)	401300.75	399370.62	403230.88
70. Number of Large Trees (Total)	200130.75	198100.75	202160.75
71. Carbon sequestration (Total)	401300.75	399370.62	403230.88
72. Standing timber (Total)	401300.75	399370.62	403230.88
73. Number of Large Trees (Total)	200130.75	198100.75	202160.75
74. Carbon sequestration (Total)	401300.75	399370.62	403230.88
75. Standing timber (Total)	401300.75	399370.62	403230.88
76. Number of Large Trees (Total)	200130.75	198100.75	202160.75
77. Carbon sequestration (Total)	401300.75	399370.62	403230.88
78. Standing timber (Total)	401300.75	399370.62	403230.88
79. Number of Large Trees (Total)	200130.75	198100.75	202160.75
80. Carbon sequestration (Total)	401300.75	399370.62	403230.88
81. Standing timber (Total)	401300.75	399370.62	403230.88
82. Number of Large Trees (Total)	200130.75	198100.75	202160.75
83. Carbon sequestration (Total)	401300.75	399370.62	403230.88
84. Standing timber (Total)	401300.75	399370.62	403230.88
85. Number of Large Trees (Total)	200130.75	198100.75	202160.75
86. Carbon sequestration (Total)	401300.75	399370.62	403230.88
87. Standing timber (Total)	401300.75	399370.62	403230.88
88. Number of Large Trees (Total)	200130.75	198100.75	202160.75
89. Carbon sequestration (Total)	401300.75	399370.62	403230.88
90. Standing timber (Total)	401300.75	399370.62	403230.88
91. Number of Large Trees (Total)	200130.75	198100.75	202160.75
92. Carbon sequestration (Total)	401300.75	399370.62	403230.88
93. Standing timber (Total)	401300.75	399370.62	403230.88
94. Number of Large Trees (Total)	200130.75	198100.75	202160.75
95. Carbon sequestration (Total)	401300.75	399370.62	403230.88
96. Standing timber (Total)	401300.75	399370.62	403230.88
97. Number of Large Trees (Total)	200130.75	198100.75	202160.75
98. Carbon sequestration (Total)	401300.75	399370.62	403230.88
99. Standing timber (Total)	401300.75	399370.62	403230.88
100. Number of Large Trees (Total)	200130.75	198100.75	202160.75

Numerical output with objective values and assignment of management options to cells



Radar chart with range of indicator values and optimization result



Distribution of management options in the CSR Catalonia

